Lecture 21

AoI and Sampling

Reading: Wait or Update TIT 2017.

JSAC AoI survey

Sun, Cyr 2019.

Ornee, Sun 2020

Key concepts:

1. fractional programing.

2. sufficient statistics.

3. convex optimization.

4. (7) has a unique root. Why?

β is the root of.

$$E\left[\int_{D_i(\beta)}^{D_{i+1}(\beta)} P(\Delta(t))\,dt\right] - \beta E\left[D_{i+1}(\beta) - D_i(\beta)\right] = 0 \qquad (7).$$

$$h(\beta) = \inf_{\pi\in\Pi_1} E\left[q_r(Y_j, Z_j, Y_{j+1})\right] - \beta E\left[Y_i + Z_i\right]$$

$$= E\left[\int_{D_i(\beta)}^{D_{i+1}(\beta)} P(\Delta(t))\,dt\right] - \beta E\left[D_{i+1}(\beta) - D_i(\beta)\right]$$

$$h(\beta) = 0 \qquad (8).$$

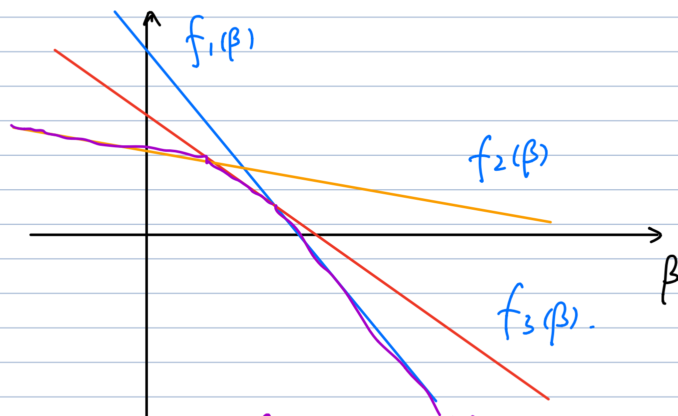
Let $L(\beta,\pi) = E\left[q_r(Y_j, Z_j, Y_{j+1})\right] - \beta E\left[Y_i + Z_i\right]$

$L(\beta,\pi)$ is linear & strictly decreasing in β.

Hence,

$$h(\beta) = \inf_{\pi \in \overline{\pi}_1} L(\beta, \pi).$$

the infimum of linear & strictly decreasing functions is concave & strictly decreasing.



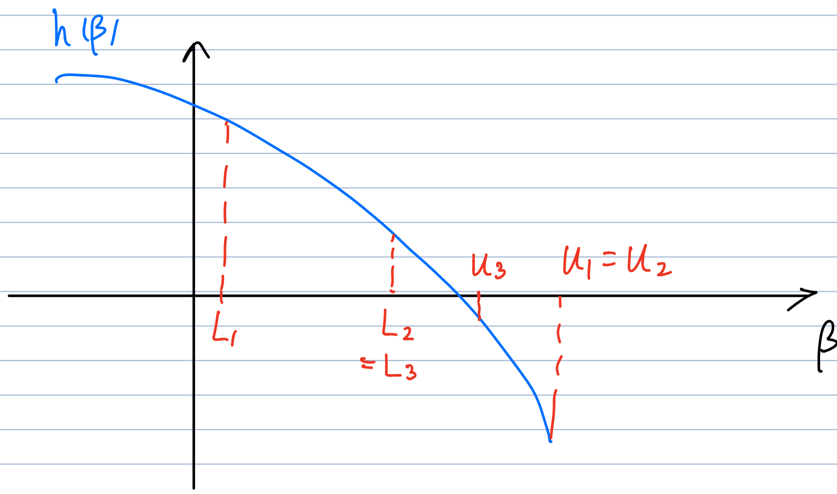
$g(\beta) = \min\{f_1(\beta), f_2(\beta), f_3(\beta)\}$

$g(\beta)$ is concave & strictly decreasing.

$h(\beta)$ is concave & strictly decreasing

$h(\beta) = 0$ has a unique root.

Algorthm 1. bisection search.



reduce by half in each iteration.

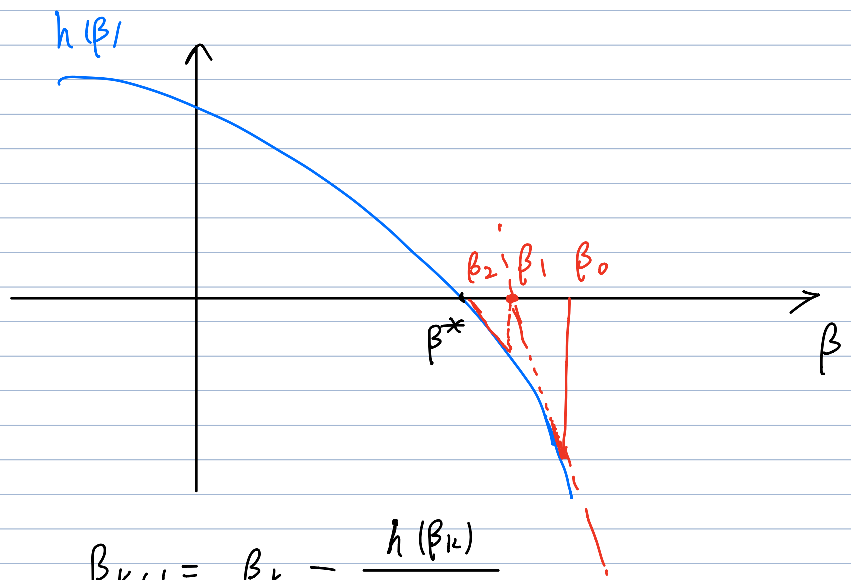
Def: $e_k = U_k - L_k$.

$$e_{k+1} = \frac{1}{2} e_k .$$

linear convergence rate

$$\left| \frac{e_{k+1}}{e_k} \right| = \frac{1}{2} .$$

Alg 2. Netwon's method



$$\beta_{k+1} = \beta_k - \frac{h(\beta_k)}{h'(\beta_k)}.$$

if $\beta_0 > \beta^*$, then

β_k is a decreasing sequence.

converges quickly.

$$|\beta_{k+1} - \beta^*| \leq M \, |\beta_k - \beta^*|^2.$$

Locally quadratic convergence rate.

$$\frac{|e_{k+1}|}{|e_k|^2} \leq M.$$

Alg 3. Fixed point iteration.

$$h(\beta) = E\left[\int_{D_i(\beta)}^{D_{i+1}(\beta)} P(\Delta(t))\, dt\right] - \beta\, E\left[D_{i+1}(\beta) - D_i(\beta)\right]$$

Def.

$$g(\beta) = \frac{E\left[\int_{D_i(\beta)}^{D_{i+1}(\beta)} P(\Delta(t))\, dt\right]}{E\left[D_{i+1}(\beta) - D_i(\beta)\right]}$$

$$h(\beta) = 0 \iff g(\beta) = \beta.$$

$$\beta_{k+1} = g(\beta_k).$$

if $\beta_0 \geq \beta^*$, then

β_k is a decreasing sequence.

$$|\beta_{k+1} - \beta^*| \leq M\, |\beta_k - \beta^*|^2.$$

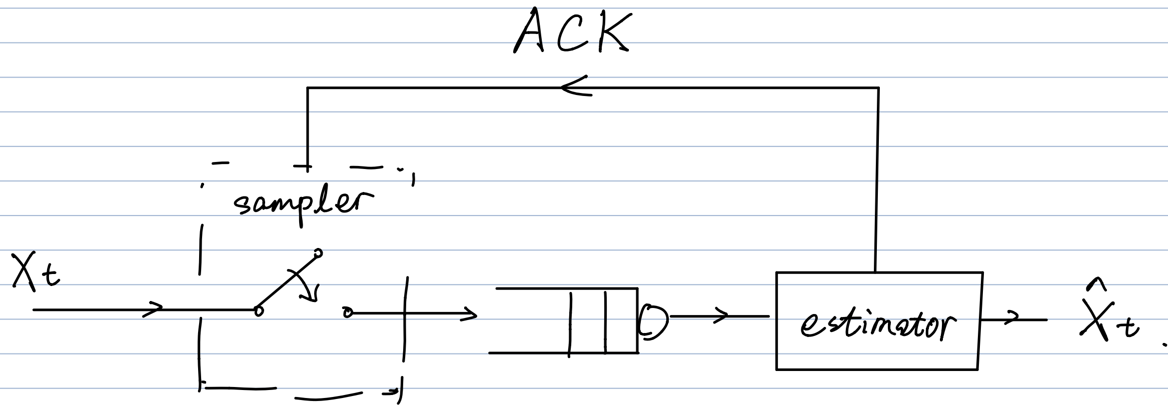
Locally quadratic convergence rate.

$$\frac{|e_{k+1}|}{|e_k|^2} \leq M.$$

Reading: Lemma 2 & Section III A-1.

Ornee, Sun 2020.

Simple-Aware Sampling:



sample i: generation time S_i delivery time D_i

$$\Delta(t) = t - \max\{S_i : D_i \le t\}.$$

ACK: zero feedback delay.

server idle/busy state is known at the sampler.

- Consider a FCFS queueing system, with a general service time distribution.
- service time. Y_i. i.i.d.

$$0 < E[Y_i] < \infty.$$

- $\hat{X}_t$: MMSE estimator.

$$mse_{opt} = \inf_{\pi \in \Pi} \limsup_{T \to \infty} \frac{1}{T} E\left[\int_0^T (X_t - \hat{X}_t)^2 dt\right]$$

$\pi = (S_1, S_2, \ldots)$: sampling policy.

Π: the set of causal sampling policies.

Stopping time:

Example:

First raining day of March 2021.

the N th day.

N is determined by a process X_t.

$X_t \in \{$rain, sunny, snow$\}$.

N is a random time.

$N = n$ is determined by $\{X_1, X_2, \ldots, X_n\}$ for all n.

Each S_i is a stopping time of X_t & feedback process

Thm:

If X_t is a Wiener process. then

$(S_1(\beta), S_2(\beta), \cdots)$ is an optimal solution, where

$$S_{i+1}(\beta) = \inf\{ t \geq D_i(\beta) : |X_t - \hat{X}_t| \geq v(\beta) \}.$$

where β is the unique root of

$$E\left[\int_{D_i(\beta)}^{D_{i+1}(\beta)} (X_t - \hat{X}_t)^2 dt \right] - \beta E[D_{i+1}(\beta) - D_i(\beta)] = 0,$$

in addition, $\beta = mse_{opt}$.

$$v(\beta) = \sqrt{3(\beta - E(Y_i))}.$$

For other Markov processes. the function $v(\beta)$ varies.

Reading: Section VI-B.

AoI survey 2021.

AoI & signal-agnostic sampling:

If (i) X_t is a time-homogenous Markov Chain.

(ii) the sampling times S_i are independent of X_t.

then there exists an increasing function $P(\cdot)$. such that

$$E\left[\int_0^T (X_t - \hat{X}_t)^2 dt\right] = E\left[\int_0^T P(\Delta(t))dt\right]$$

For sample-agnostic sampling:

$$\inf_{\pi \in \Pi_{agnostic}} \limsup_{T \to \infty} \frac{1}{T} E\left[\int_0^T (X_t - \hat{X}_t)^2 dt\right]$$

$$= \inf_{\pi \in \Pi_{agnostic}} \limsup_{T \to \infty} \frac{1}{T} E\left[\int_0^T P(\Delta(t))dt\right]$$

$$= mse_{age\text{-}opt}.$$

Thm:

If (i) X_t is a time-homogenous Markov Chain.

(ii) the sampling times S_i are independent of X_t.

then

$(S_1(\beta), S_2(\beta), \cdots)$ is an optimal solution, where.

$$S_{i+1}(\beta) = \inf\{ t \geq D_i(\beta) : E[(X_{t+Y_{i+1}} - \hat{X}_{t+Y_{i+1}})^2] \geq \beta \},$$

where β is the unique root of

$$E\left[\int_{D_i(\beta)}^{D_{i+1}(\beta)} (X_t - \hat{X}_t)^2 dt\right] - \beta E[D_{i+1}(\beta) - D_i(\beta)] = 0,$$

in addition, $\beta = mse_{age\text{-}opt}$.

Reading: Section VI-B.

AoI survey 2021.